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Sensitivity Analysis of Flexible Multibody Dynamics Applied to the Gradient-based Design Optimization of a Tyrolean Weir Cleaning Mechanism

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EXTENDED ABSTRACT

1 Introduction

This work introduces efficient design sensitivity analysis based on a semi-analytical approach of flexible multibody systems (FMBS) with the floating frame of reference formulation (FFRF) for gradient-based design optimization of a Tyrolean weir cleaning mechanism. With FFRF, the dynamic simulation is decoupled from the finite-element (FE) model by means of inertia shape integrals or invariants [1]. In analogy, the here introduced design sensitivity analysis method allows for a decoupling from the FE model by the sensitivities of the invariants [2]. The governing equations with FFRF and their system parameters are differentiated analytically with direct differentiation. The partial sensitivities of the invariants are evaluated before the dynamic simulation and are computed numerically here for generality with respect to (w.r.t.) the design variables, leading to a semi-analytical method. This semi-analytical approach and the decoupling from the FE model guarantee high efficiency of the sensitivity computations and allow for efficient design optimization of FMBS, while maintaining low implementation effort and generality w.r.t. the design variables considered.

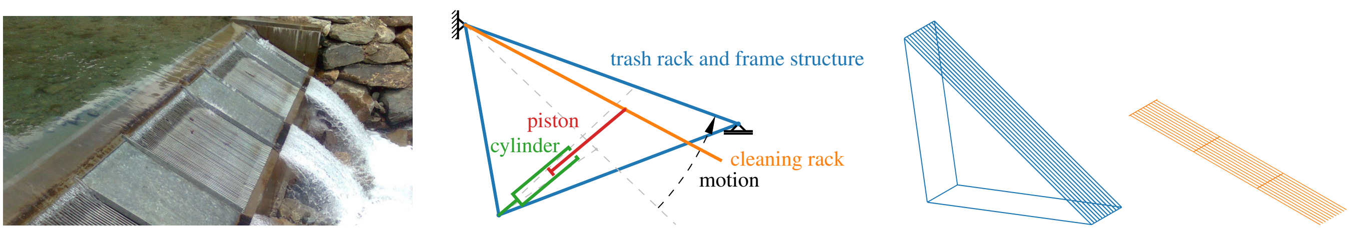
The developed method is applied to the design optimization of a Tyrolean weir cleaning mechanisms. Tyrolean weirs are drop intakes of hydroelectric power plants suitable for rivers with steep slope (Fig. 1a). A trash rack is installed at the bottom of the river behind a weir that enables waterflow while preventing particles from entering into the plant. The cleaning mechanism consists of a cleaning rack that is pushed from below by a hydraulic cylinder, lifting the particles that may get stuck between the trash rack which are then washed away by the water flow (Fig. 1b). The analysis and optimization of Tyrolean weir cleaners with rigid multibody dynamics (RMBD) and hydraulics was introduced in [3, 4] and is extended here to FMBS with the lightweight design approach combining the fields of multibody dynamics, structural analysis and hydraulics.

2 Flexible multibody dynamics

The dynamic simulation of the FMBS is carried out through a three-block solution scheme [4]: governing equations, time integration and nonlinear solver. The governing equations are expressed in the residual form of index-1 differential algebraic equations (DAE-1) including Baumgarte stabilization,

$$\underline{R} = \begin{bmatrix} \underline{\underline{m}} & {}^{q}\underline{\underline{\mathrm{J}\Phi}}^{\mathsf{T}} \\ {}^{q}\underline{\underline{\mathrm{J}\Phi}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{\ddot{q}} \\ \underline{\lambda} \end{bmatrix} - \begin{bmatrix} \underline{Q}_{\mathrm{e}} + \underline{Q}_{\mathrm{v}} - \underline{\underline{d}}\,\underline{\dot{q}} - \underline{\underline{k}}\,\underline{q} \\ \underline{Q}_{\mathrm{c}} - 2\alpha_{\mathrm{B}}\left({}^{q}\underline{\underline{\mathrm{J}\Phi}}\,\underline{\dot{q}} + {}^{t}\underline{\underline{\partial\Phi}}\right) - \beta_{\mathrm{B}}^{2}\underline{\Phi} \end{bmatrix} = \underline{0}, \tag{1}$$

with residual R, positions q, velocities $\dot{q}$, accelerations $\ddot{q}$, Lagrange multipliers λ, mass m, damping d, stiffness k, external force Q_{e}, quadratic velocity force Q_{v}, constraints Φ, Jacobian of constraints ${}^{q}\mathrm{J}\Phi$, right hand side of accelerations constraints Q_{c} and the Baumgarte stabilization coefficients α_{B}, β_{B}. Single underlines denote vectors and double underlines denote matrices. The transient solution is performed with a predictor–corrector scheme of generalized-α time integration and the nonlinearities are solved with Newton-Raphson iterations. The flexibility of the bodies is modeled with FFRF, where the position and orientation of a body reference frame is superimposed by flexible deformations of FE nodes [1]. The FFRF system parameters depend partly on position and velocity coordinates as well as on the FE model and are parametrized by inertia shape integrals or invariants [1]. The invariants are evaluated before the dynamic simulation and remain constant in time, allowing for a decoupling of the dynamic simulation from the FE model.



(a) River installation (www.guflermetall.it) (b) Schematic of flexible multibody system (c) Three-dimensional beam model of flexible bodies

Figure 1: Tyrolean weir cleaning mechanism

3 Design sensitivity analysis

Analytical sensitivity analysis is the driving motor for efficient design optimization of FMBS. These include the direct differentiation method and the adjoint variable method [5]. The former is used here through the three-block solution scheme because of the limited number of design variables and for avoiding the backward integration of the adjoint system [6]. The direct differentiation of the governing equations leads to the sensitivity equation given by

$$\underline{\underline{\nabla R}} = \begin{bmatrix} \underline{\underline{m}} & \underline{\underline{{}^{q}\mathrm{J}\Phi^{\mathsf{T}}}} \\ \underline{\underline{{}^{q}\mathrm{J}\Phi}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{\underline{\nabla \ddot{q}}} \\ \underline{\underline{\nabla \lambda}} \end{bmatrix} - \begin{bmatrix} \underline{\underline{Q}}_{\mathrm{a,pseudo}} \\ \underline{\underline{Q}}_{\mathrm{c,pseudo}} \end{bmatrix} = \underline{\underline{0}}, \tag{2}$$

with the pseudo load $\underline{\underline{Q}}_{\mathrm{a,pseudo}}$, $\underline{\underline{Q}}_{\mathrm{c,pseudo}}$ containing the sensitivities of the system parameters,

$$\begin{bmatrix} \underline{\underline{Q}}_{\mathrm{a,pseudo}} \\ \underline{\underline{Q}}_{\mathrm{c,pseudo}} \end{bmatrix} = \begin{bmatrix} \underline{\underline{\nabla Q}}_{\mathrm{e}} + \underline{\underline{\nabla Q}}_{\mathrm{v}} - \underline{\underline{\underline{\nabla d}}}\,\underline{\dot{q}} - \underline{\underline{d}}\,\underline{\underline{\nabla \dot{q}}} - \underline{\underline{\underline{\nabla k}}}\,\underline{q} - \underline{\underline{k}}\,\underline{\underline{\nabla q}} \\ \underline{\underline{\nabla Q}}_{\mathrm{c}} - 2\alpha_{\mathrm{B}} \left(\underline{\underline{\underline{\nabla {}^{q}\mathrm{J}\Phi}}}\,\underline{\dot{q}} + \underline{\underline{{}^{q}\mathrm{J}\Phi}}\,\underline{\underline{\nabla \dot{q}}} + \underline{\underline{\nabla^{t}\partial\Phi}} \right) - \beta_{\mathrm{B}}^{2} \underline{\underline{\nabla \Phi}} \end{bmatrix} - \begin{bmatrix} \underline{\underline{\underline{\nabla m}}} & \underline{\underline{\underline{\nabla {}^{q}\mathrm{J}\Phi^{\mathsf{T}}}}} \\ \underline{\underline{\underline{\nabla {}^{q}\mathrm{J}\Phi}}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{\ddot{q}} \\ \underline{\lambda} \end{bmatrix}. \tag{3}$$

The partial derivatives of the system parameters are differentiated analytically by a further step and the expressions are derived in [2]. These include the sensitivities of the invariants that are evaluated before the time integration and allow the decoupling of the dynamic simulation from the FE model. This decoupling guarantees high efficiency of the here developed sensitivity method. The direct differentiation is continued to generalized-α time integration and Baumgarte stabilization.

4 Tyrolean weir cleaning mechanism

The lightweight design approach is applied to the gradient-based design optimization of Tyrolean weir cleaners. Fig. 1b shows the multibody system. Fig. 1c shows the flexible bodies, modeled with three-dimensional Euler-Bernoulli beam elements. The mechanism is actuated by a hydraulic cylinder consisting of two rigid bodies driven by the hydraulic force resulting from the hydraulic system. The lightweight design optimization formulation is the minimization of the structural mass (objective function) with the maximum stress in the flexible bodies as design constraints and the cross-sectional dimensions of the beam elements as design variables. For manufacturing reasons, the components have constant cross sections and are parametrized with eight design variables. A maximum stress constraint over the entire motion of the mechanism is approximated with the Kreisselmeier–Steinhauser (KS) function [7] for each element to get a derivable function. The first-order optimization algorithm method of moving asymptotes (MMA) [8] is used and the optimization converges after twelve iterations. Fig. 2 shows the KS approximations of the maximum stress in the flexible bodies during the mechanism motion at the optimal design. The limit value of 100 MPa is reached in several elements on both the trash rack with frame structure and the cleaning rack.

(a) Trash rack with frame structure

(b) Cleaning rack

Figure 2: Maximum stress of the flexible bodies at optimal design

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